A-6

Q.5 Suppose that, a tumour in a rat is approximately spherical and its rate of growth is proportional to its diameter. If the tumour has diameter 5 m.m., when detected at 8mm, after three months later, what will the diameter be after another three months?

(a) 10.1 m.m. (b) 12.8 m.m. (c) 11.6 m.m. (d) 13.2 m.m.

Sol^n :- toumor is in 3-D spherical shape so rate of growth is change in volume. with time.

$$\frac{dV}{dt} \propto 2r \qquad (2r = \text{diameter})$$

$t = 0 \Rightarrow d = 5$ m.m.

$t = 3 \rightarrow d = 8$ m.m.

$t = 6 \Rightarrow d = ?$

$$\frac{dv}{dt} = 2kr \quad \text{—(i)} \quad (k = \text{proportionality constant})$$

$\because \; V = \frac{4}{3}\pi r^3$

$$\frac{dv}{dt} = \frac{4}{\cancel{3}}\pi \cdot \cancel{3}r^2\frac{dr}{dt}$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt} \quad \text{—(ii)}$$

So from (i)

$$\cancel{4}\pi r^{\cancel{2}}\frac{dr}{dt} = \cancel{2}kr \Rightarrow r^2\frac{dr}{dt} = \frac{2k}{4\pi}r$$

~~2πr/dr / k/kdt~~

$$r^2\frac{dr}{dt} = c_1 r \qquad \left(\text{let } c_1 = \frac{2k}{4\pi}\right)$$

$$\int r\,dr = c_1\int dt$$

$$\Rightarrow \frac{r^2}{2} = c_1 t + c_2$$

$$\Rightarrow \frac{d^2}{8} = c_1 t + c_2$$

$$\Rightarrow \frac{d^2}{8} = c_1 t + c_2$$

at $t=0$ $d=5$ at $t=3, d=8$

$$\boxed{\frac{25}{8} = c_2}$$

$$\frac{64}{8} = 3c_1 + \frac{25}{8}$$

$$\boxed{c_1 = \frac{13}{8}}$$

$$\frac{d^2}{8} = \frac{13}{8}t + \frac{25}{8}$$

$$\Rightarrow d^2 = 13t + 25$$

at $t = 6 \Rightarrow d^2 = 103$

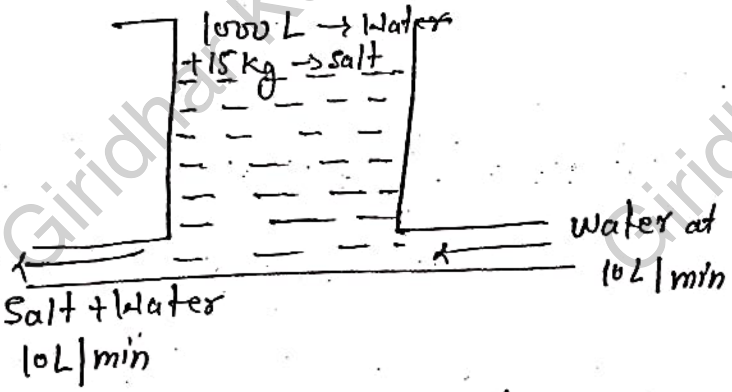
$$\boxed{d \approx 10.1 \, m.m.}$$ Ans

A-6
Q.6 A tank holds 1000 liters of water, in which 15 kg of salt is dissolved. Pure water enter the tank at the rate of 10 liters/minutes. The solution is kept throughout mixed and drained from the tank at the same rate. If 'm' is the mass of the salt in the tank at any time 't' which of the following options decribes the rate of change of the mass of the salt in the tank?

Sol^n

$t = 0 \Rightarrow 15$ kg

$t = t \Rightarrow m$ kg



$\therefore \frac{dm}{dt} \propto$ Amount of salt getting out of the tank per minute

In 1000 L $\longrightarrow$ m kg

10 L $\longrightarrow$ $\frac{m}{1000} \times 10$ kg $= \frac{m}{100}$ kg

$$\frac{dm}{dt} \propto \frac{m}{100}$$

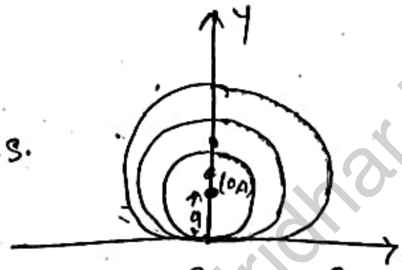
dm = − m

$$\boxed{\frac{dm}{dt} = -\frac{m}{100}}$$ Ans

N-0
Q.8 The differential equation of all circles passing through the origin and having their centers on the y-axis

Sol^n Differential equation of circles means solution of this ~~eq~~ differential equation gives the family of circles.

D.E.
$\downarrow$
family of circles.



Note:- The number of variable parameter will be the order of corresponding differential equation.

$x^2+(y-a)^2=a^2$
$\downarrow$
1st order D.E

∴ Here we change only one parameter (radius) then center is automatically change and we get family of curve having centers on the y-axis and ~~center~~ passing through the origin. So Here order of differential equation is one.

$$x^2+(y-a)^2=a^2$$

$$\Rightarrow \cancel{2}x + \cancel{2}(y-a)\frac{dy}{dx} = 0$$

$$\Rightarrow (y-a)\frac{dy}{dx} = -x$$

$$\Rightarrow \boxed{(y-a) = -\frac{x}{dy/dx}}$$

$$\Rightarrow \boxed{a = y + \frac{x}{dy/dx}}$$

$$x^2 + \frac{x^2}{\left(\frac{dy}{dx}\right)^2} = \left[y + \frac{x}{\left(\frac{dy}{dx}\right)}\right]^2$$

$$\Rightarrow x^2 - y^2 = \frac{x^2}{\left(\frac{dy}{dx}\right)^2} - \frac{x^2}{\left(\frac{dy}{dx}\right)^2} + \frac{2xy}{\left(\frac{dy}{dx}\right)}$$

$$\Rightarrow x^2 - y^2 = \frac{2xy}{(dy/dx)}$$

$$\Rightarrow \boxed{(x^2-y^2)\left(\frac{dy}{dx}\right) = 2xy}$$

* Case 3:- Linear Differential Equation :-

A differential equation is said to be linear if the power of the dependent variable and it's derivatives are one.

First Order Linear Differential Equation :-

Standard form of Linear Differential Equation of first order. is -

$$\boxed{\frac{dy}{dx} + P(x)y = Q(x)}$$

Here $P(x)$ and $Q(x)$ are either constants or function of x.

$$\boxed{I.F. = e^{\int P(x)dx}}$$

$$\underbrace{(I.F)\frac{dy}{dx} + (I.F.)P(x)y}, = (I.F.)Q(x)$$

$$\Rightarrow \frac{d}{dx}[y(I.F.)] = (I.F)Q(x)$$

$$\Rightarrow \int d[y(I.F.)] = \int (I.F.)Q(x)dx$$

Short. Trick.

$$\boxed{y(I.F.) = \int (I.F.)Q(x)dx}$$

If $\boxed{\frac{dx}{dy} + P(y)x = Q(y)}$

$$\boxed{I.F. = e^{\int P(y)dy}}$$